System Resonance Frequency Analysis With Distributed Parameter Cylinder Models

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During the working stroke of hydraulic cylinder drives unexpected and unwanted resonances in attached pipes are often unavoidable. A main reason is the continuous change of the system's natural frequency because of variable piston and cylinder positions. An analytical investigation of variable resonance situations is difficult since geometric boundary conditions like e.g. diameters and lengths of pipes/cylinders as well as nonlinear effects like e.g. the fluid's compressibility or a viscous-elastic tube expansion must be considered.
Typically, concentrated parameter models are used for cylinder drive simulations, though such models are not capable to represent the exact influence of variable cylinder chamber volumes on the resonance situation. This publication presents a new approach that realizes a variable cylinder chamber volume or length in combination with a advanced distributed parameter approach. With theoretical fundamental investigations as well as a simplified example it is shown, that by means of the distributed parameter cylinder, it is possible for the first time to analyse the oscillation situation of a cylinder drive during the complete operating cycle.

Keywords: simulation, pressure wave, hydraulic cylinder, pipe, resonance
Target audience: Simulation, Mobile Hydraulics, Design Process

1 Introduction

Since Blaise Pascal established 1647-1648 the "principle of transmission of fluid-pressure" (Pascal's law), hydraulic systems found their way into a wide range of applications. This includes international pipelines, mobile applications (airplanes, construction machines or automobiles) as well as stationer systems (hydraulic press or indoor water/heating installations). Hydraulic systems feature a high power density and a robust design, so that they are predestined for mobile applications. A natural enemy of hydraulic systems are uncontrollable pressure oscillations at system resonance frequencies. Previous investigations showed that already minimal system excitations in the region of system resonance frequencies result in severe and unfavourable pressure oscillations /12/. High pressure or flow oscillations result in multiple failure modes:

- Noise/NVH (oscillation of walls or housings)
- control issues (cylinder positioning or controller oscillation)
- part destruction (high peak pressure)

To achieve a time and cost-efficient system development process it is mandatory to analyse critical system states already early in the development. Beneficial is previous first prototypes are build. Simple systems like a single pipe are simply analysed with fundamental theoretical equations. Unfortunately, nearly all practical systems are more complex and have to be analysed by simulation models. In addition to the basic system complexity, transient system changes and different control strategies are important parts of a detailed and meaningful system investigation. Multiple applications proved that in most cases, a one-dimensional simulation model is adequate and a perfect compromise between model accuracy and simulation time /1,3,8/. A complete system analysis comprises variations in excitations as well as system structural changes. Main system excitations are generated by pumps, valves or external forces on cylinders. Especially pump frequencies vary during different use cases like system start-up and stop or in modern pump driven cylinder drives. In addition, the system resonance frequencies change due to moving cylinders or dur to switched vales, that (de)couple sub-systems.

Previous publications showed that a meaningful system analysis and layout requires an accurate pressure wave calculation inside pipes, with consideration of static and dynamic friction as well as wall pumping /3, 8/. Modern pipe models based on extended method of characteristics are fully capable to predict system pressure oscillation if all pipe lengths are constant during the simulation /1,2,3,4,5,6/. Unfortunately, this requirement cannot be fulfilled for all applications. For example continental pipelines expand and shrink as function of temperature (day/night, summer/winter) or cylinder move. Each hydraulic cylinder with a maximal length significant bigger than its diameter can be considered as a pipe with variable length. Consequently, a correct wave calculation in pipes with variable length is necessary to guarantee an accurate system analysis.

2 Test Case

A fundamental investigation based on a complex simulation model like shown in figure 1 (left) is not meaningful, because multiple effects are mixed and no easy theoretical solution is available. Thus, a simplified test case, composed of an adjustable pump, a pipe and a cylinder is chosen. The pump generates pressure pulsations with frequencies proportional to its current speed and the cylinder movement constantly changes the system's resonance frequencies (figure 1).

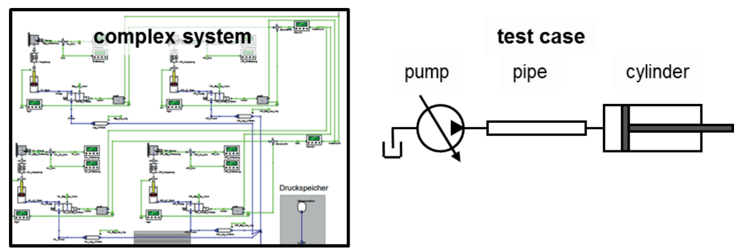

Figure 1: Simplified test case.

Two alternative set-ups of the system are shown in figure 2. In the first case, the cylinder features a diameter much higher than the supply pipe ($d_{cylinder} >> d_{pipe}$). This set-up features a pipe that is connected to a large reservoir. The other test case presents a set-up where cylinder and pipe have the same diameter. This is a good choice for theoretical analysis, because the system's resonance frequency is easy to calculate. This report focus on both test cases, assuming that real applications will be between.

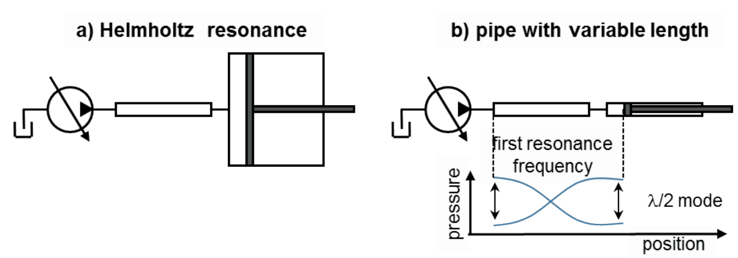

Figure 2: Possible test cases

2.1 Pump Pulsation

To enable a flexible and clearly defined system excitation, an idealized 9-piston pump is used (figure 3). Beside the volumetric displacement, also the decompression effect, known from positive displacement pumps, is introduced.

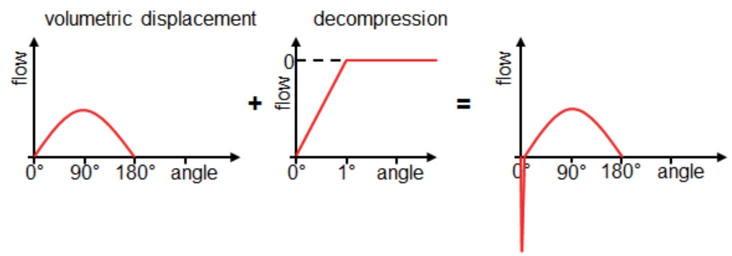

Figure 3: pump flow parts of one cylinder

The superposition of nine flow patterns and succeeding PT1-smoothing results in an ideal pump flow shown in figure 4.

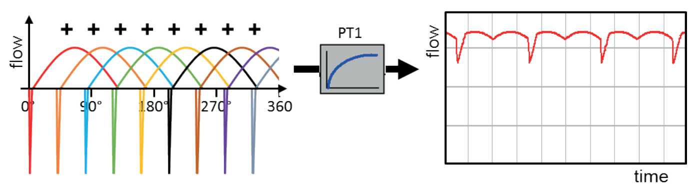

Figure 4: idealized pump flow

In comparison figure 5 shows on the left-hand side a typical measured flow characteristic as well as on the right-hand side a typical measured decompression effects /7, 9/.

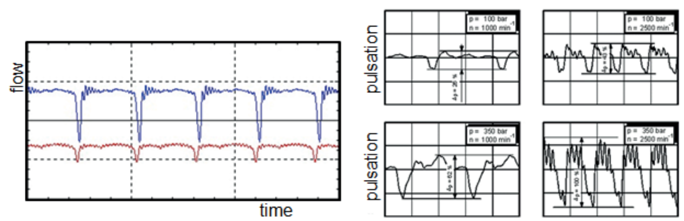

Figure 5: real pump flow /7/ and measured decompression effects /9/

Finally, figure 6 shows the frequency analysis of the characteristic pump flow at 1500 rpm and Campbell-diagram of the pump flange pressure during a run-up of the pump from 0 to 1500 rpm.

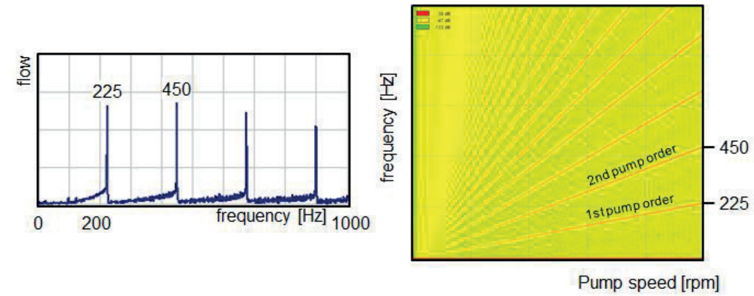

Figure 6: frequencies of pump flow at 1500 rpm an Campbell-diagram of a pump run-up

2.2 Pump Speed Variation and Expected Resonance Frequencies

The cylinder-model's capability to predict the correct system frequencies is analyzed by a slow cylinder movement, which is for example used for positioning tasks, extruders or for a hydraulic press. At simulation start, the pump speed increases to standard speed of n = 1500 rpm (figure 7, blue). Afterwards, the pump speed is unchanged and the cylinder moves out at continues speed (figure 7, red). To allow an exact positioning of the cylinder the pump speed is slowly reduced until zero. This results in a cylinder speed reduction until the end position is reached.

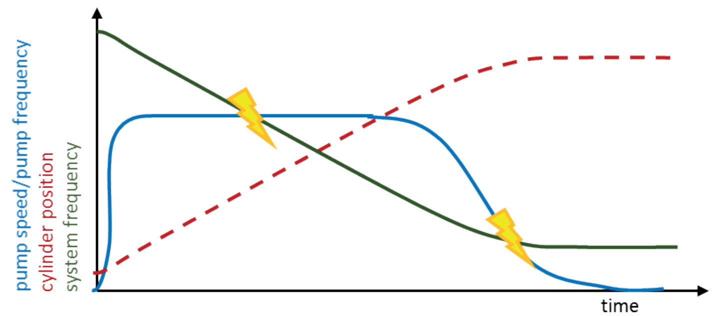

Figure 7: simulation test case

Since the system excitation frequency is proportional to the pump speed, the blue line in figure 7 can also be interpreted as excitation frequency. The resonance frequency at simulation start is higher than the first pump frequency. During the fast pump speed ramp-up, no critical states are expected (short excitation duration). Afterwards the cylinder moves out, what increases the total pipe length and consequently reduces the system's resonance frequency. At a specific point in time, it is unavoidable that the system resonance frequency is similar to the first pump frequency, which will lead to a critical system resonance. Afterwards the first system resonance frequency is below the pump frequency. This changes, if the pump speed drops and the excitation frequency consequently lowers to the present system resonance frequency. The following investigation focuses on the first resonance frequency only. In reality and in the simulation higher frequency orders will have a similar effect onto the system.

3 SIMULATION

For simplification, typical 1D simulation approaches subdivide the system shown in figure 2b into components, which are approximated with physical fundamental equations or measurement data.

3.1 concentrated parameter models

A widely distributed method for pipe simulation is dividing the pipe into multiple succeeding volumes and resistors (concentrated parameter). An additional volume approximates the cylinder. Multiple publications /1,2,3,4,5,6,10/ showed that classical concentrated parameter models are not sufficient to accurate predict pressure waves in an acceptable calculation time. Thus, this investigation skips classical pipe models and focus on modern and more accurate model types based on the method of characteristics with mathematical extensions for high frequency dynamic friction /2,3/. In contrast to the concentrated parameter model, this approach is called distributed parameter model.

3.2 mixed models (concentrated and distributed)

Figure 8 shows a first simulation model of the investigated test system (figure 2b) with a distributed parameter pipe model and a classical cylinder model. The implemented pump model generates the proposed volumetric flow pulsation shown in figure 4.

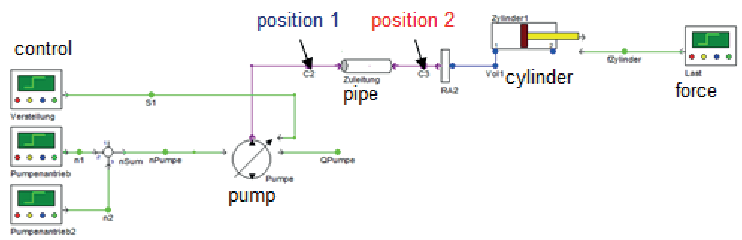

Figure 8: model A with distributed parameter pipe and classical cylinder

The cylinder is exposed to a constant force during the simulation and comprises internal friction. Table 1 specifies some fundamental model parameter:

Table 1: fundamental model parameter

parameter	vale	unit
pump displacement	125	ccm3
pump cylinder	9	-
pipe length	200	mm
pipe diameter	30	mm
cylinder length	5000	mm
cylinder diameter	30	mm
cylinder start position	200	mm
cylinder weight	4000	kg

Figure 9 shows multiple calculated system responses, which allow a detailed analysis of the system behaviour and especially system resonance frequencies and states. Beside the pump speed and cylinder movement, the pressure and flow oscillation at position 1 and 2 are important for an analysis. After first start-up effects during the first second, an increased and critical pressure/flow oscillation occurs at 3.3 and 18.8 seconds. Especially at these times, the spectrogram in figure 9 (right) shows an increased effect at 1800 Hz and 1670 Hz. These frequencies correspond to the main resonance frequencies found with a fast Fourier transform [FFT] in the pressure signal.

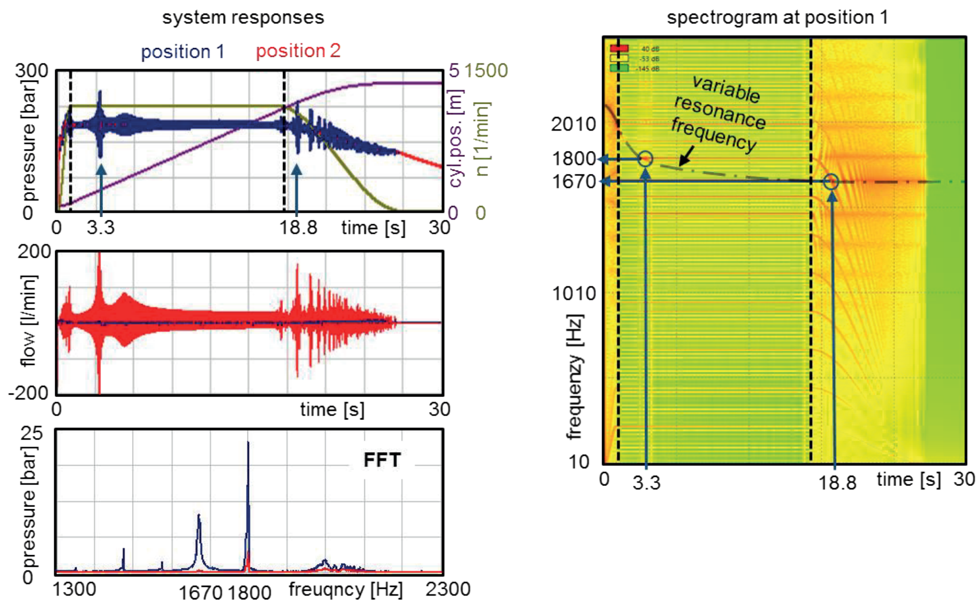

Figure 9: responses and analysis of model A

In the next step, it is necessary to calculate the expected system resonance frequency at t = 3.3 and 18.8 seconds. The test case is set-up in a way ($D_{pipe} = D_{cylinder}$) that the theoretical resonance frequency is a λ/2 mode like shown in figure 10. With a speed of sound $c \approx 1317 \frac{m}{s}$ the expected theoretical system resonance frequencies are calculated to:

Table 2: expected frequencies

time [s]	pipe length [mm]	frequency [Hz]
3.3	963.8 (200+763.8)	683.2
18.8	4077.4 (200+3877.4)	161.5

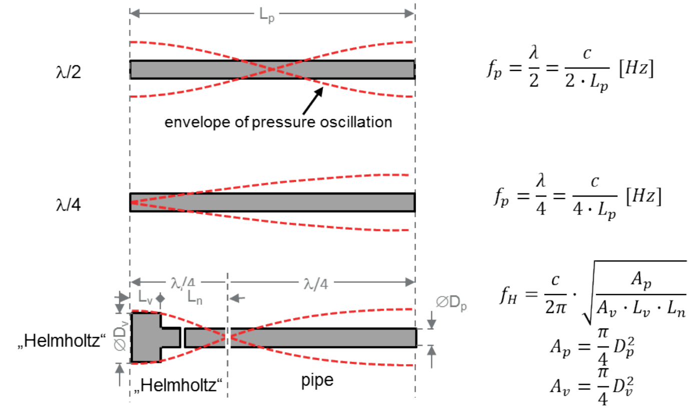

Figure 10: first oscillation modes and "Helmholtz" like frequency

Unfortunately, the expected and observed (simulation) frequencies do not fit. A closer look to the 2D pressure plot in figure 11 unveils the main calculation problem. The pressure wave calculation in the pipe is working very well with a high resolution but the state of the art cylinder approximation with concentrated parameter simplifies the cylinder system to a single volume. This volume has a single pressure and no pressure wave calculation is possible, that accounts for the length of the pipe. The characteristic of the investigated system is moved from a pipe with changing length to something like a "Helmholtz" resonator with changing volume size. To calculate the theoretical resonance frequency, we have to divide the test system into two parts with $\lambda/4$ oscillation modes like shown in figure 10. Thereby, a small part of the pipe is assigned to the resonator. During the resonance case, the frequency of both parts are the same and the following equations have to be true.

$$f_H = f_{\frac{\lambda}{4}} \quad \rightarrow \quad \frac{c}{2\pi} \cdot \sqrt{\frac{A_p}{A_v \cdot L_v \cdot L_n}} = \frac{c}{4 \cdot L_{\frac{\lambda}{4}}}$$

$$L_{sys} = L_v + L_n + L_{\frac{\lambda}{4}}$$

$$with\ A_p \equiv A_v\ \ follows$$

$$\frac{1}{\pi} \cdot \sqrt{\frac{1}{L_v \cdot L_n}} = \frac{1}{2 \cdot L_{\frac{\lambda}{4}}} \quad \rightarrow \quad \pi^2 \cdot L_v \cdot L_n = 4 \cdot L_{\frac{\lambda}{4}}^{\ 2}$$

This leads to a solution for L_n dependent on the current system and cylinder length.

$$L_n = L_{sys} - L_v + \frac{\pi^2 \cdot L_v}{8} - \frac{\pi \sqrt{\frac{L(L \cdot \pi^2 + 16 \cdot L_{sys} - 16 \cdot L_v)}{16}}}{2}$$

For our test case the frequencies in table 3 are calculated, which correspond very well to the detected simulation results. The theoretical frequencies will be a little bit greater than the real and the simulated values, because they do not consider (dynamic) friction effects. The oscillation nodes in front of the volume (distance l_n) is also visible in a 2D pressure plot (figure 11) and fit to the theoretical values. This proofs that the state of the art cylinder simplification changes the system behaviour and is not adequate for a meaningful analysis. Although the wrong system is simulated, the spectrogram in figure 2 shows that the used simulation and analysis tool is capable to simulate changing system resonance frequencies caused by volume changes.

Table 3: expected frequencies for "Helmholz" like system

time [s]	L_{sys} [mm]	L_v [mm]	L_n [mm]	frequency [Hz]
3.3	963.8	763.8	17.645	1805
18.8	4077.4	3877.4	4.015	1680

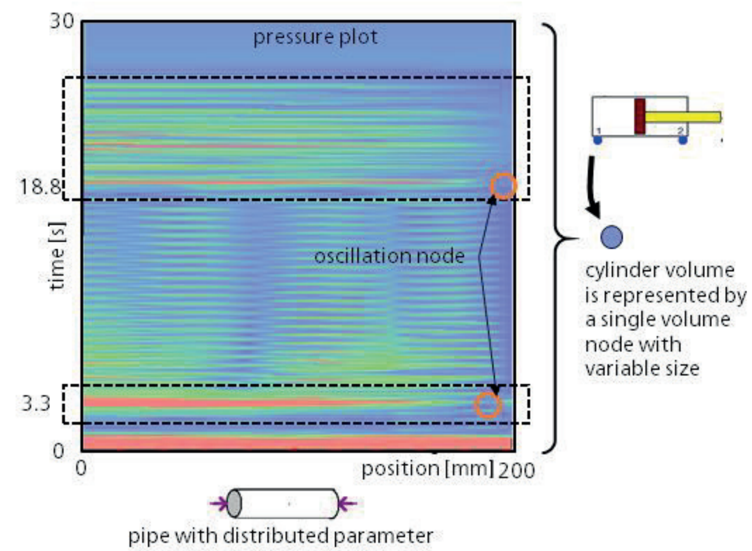

Figure 11: 2D pressure plot for model A

3.3 distributed parameter models

The investigation shows that the current test system can only be correctly analysed if the wave effects are also considered in the cylinder. An often-used approach is to replace the cylinder with a distributed parameter pipe model like shown in figure 12 or done in /11/. In this approach the system is excited with a pressure or flow frequency sweep. With this approach the resonance frequencies of a fix model is analysed. Two main drawbacks have to be considered. Each cylinder position needs a separate model parameter, calculation and analysis. In the case of multiple cylinders, the number of parameter sets and calculations rocket, because each combination has to be considered. Especially the analysis of complex systems is not efficient with this method. In addition, this approach modifies the investigated system, too. The piston wall is not moving and thus the pump has to work at a unrealistic flow conditions ($\bar{Q} = 0\ l/min$). Transient systems effects are not considered which might lead to wrong conclusions and wrong development targets.

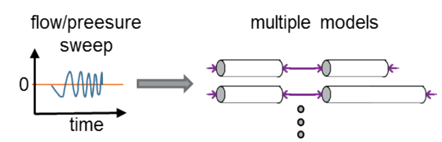

Figure 12: multiple distributed parameter pipes

3.3.1 distributed cylinder model

In order to overcome the mentioned drawbacks, all fundamental pressure wave effects have to be considered and implemented in a moving cylinder model. A promising approach is to extend the distributed pipe with a flexible length and consequently with a flexible number of elements (figure 13, A). However, adding and deleting complete pipe elements requires (de)allocating memory at every element number change and a reinitialization of the numerical grid. More convenient is an initialization directly at maximal pipe length and a subsequent cylinder position dependent (de)activation of pipe elements (figure 13, B). Activating an element comprises two tasks:

a) moving the pipe boundary condition to another element
b) initialize pressure and speed of the activated elements based on current system state

Independent from changing the number of activated elements, each piston movement results in a system pressure change, which is approximated by a flow in or out of the current last activated element. The flow Q correlates to the piston area and speed.

$$Q = A_p \cdot v_p$$

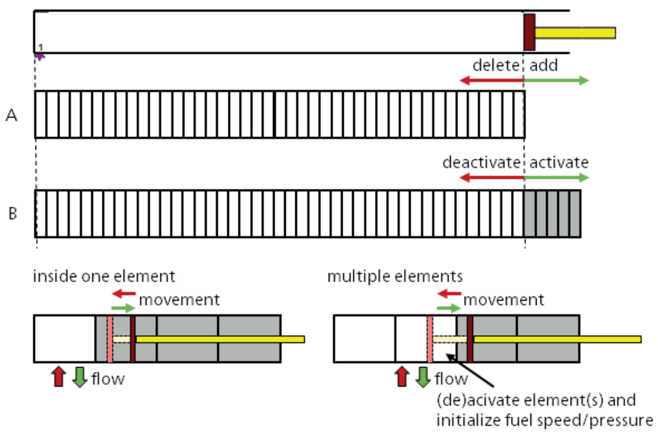

Figure 13: distributed pipe with variable length

Based on the pipe model, the easiest approach for a cylinder is to apply two parallel pipes like shown in figure 14. It is evident, that elements at a specific position along the pipe always belong only to one side of the cylinder. Thus, it is possible to set up a calculation method, which requires only one set of elements (figure 14). The activated elements of both cylinder sides can directly touch each other (with separate end conditions) or some intermediate elements can be deactivated.

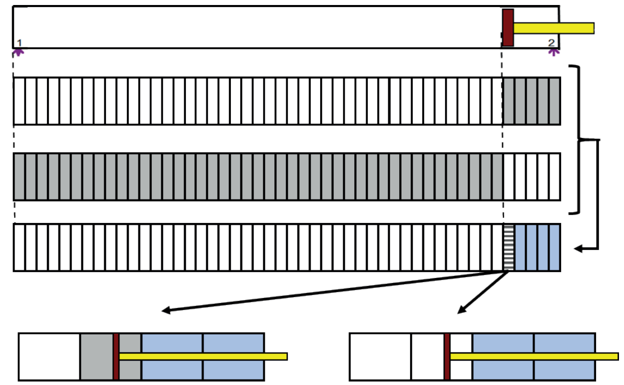

Figure 14: distributed cylinder element assignment

Figure 15 exemplarily shows the flow and element (de)activation for different typical cylinder movements if no central deactivated element is used.

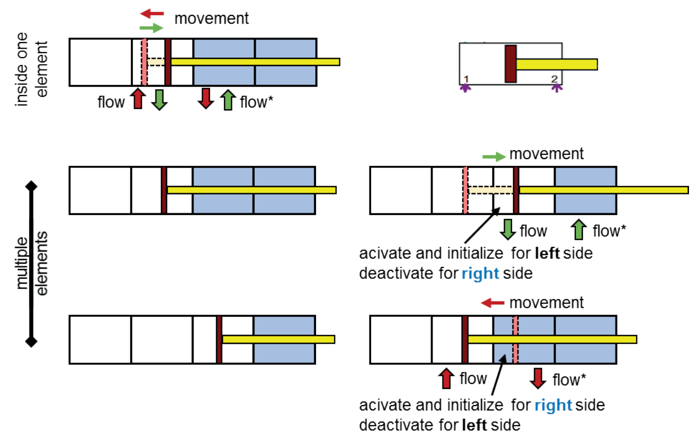

Figure 15: distributed cylinder witch deactivated intermediate element

3.3.2 simulation model B

In the test case, the distributed cylinder model replaces the concentrated cylinder model, which allows to calculate the complete system with the approach of distributed parameter (figure 16).

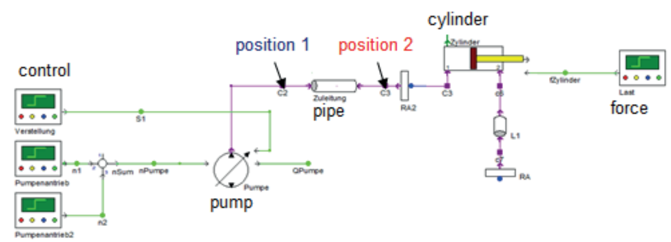

Figure 16: model B with distributed parameter pipe and cylinder

The response analysis in figure 17 shows resonance effects at t = 5.8, 13.0 and 21.3 seconds. The spectrogram reveals corresponding system resonance frequencies at 140, 225 and 450 Hz. In addition, the continuously changing system resonance frequencies and the temporal effect of the resonance on the system are identifiable.

The expected (theoretical) frequencies are shown in table 4 and correspond to the simulation results. The 2D pressure plot in figure 18 confirms the assumed $\frac{\lambda}{2}$ oscillation mode at 5.8 and 13.0 seconds. At 21.3 seconds, the effects are smaller which prevents a clear oscillation node definition.

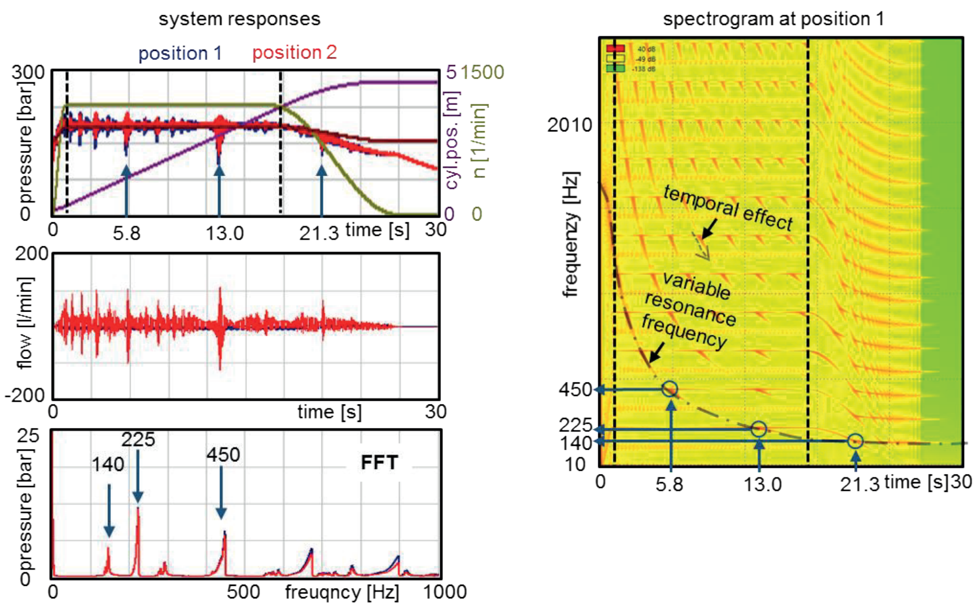

Figure 17: responses and analysis of model B

Table 4: expected frequencies, $\frac{\lambda}{2}$

time [s]	pipe length [mm]	frequency [Hz]
5.8	1467 (200+1267)	449
13.0	2910 (200+2710)	226
21.3	4455 (200+4255)	148

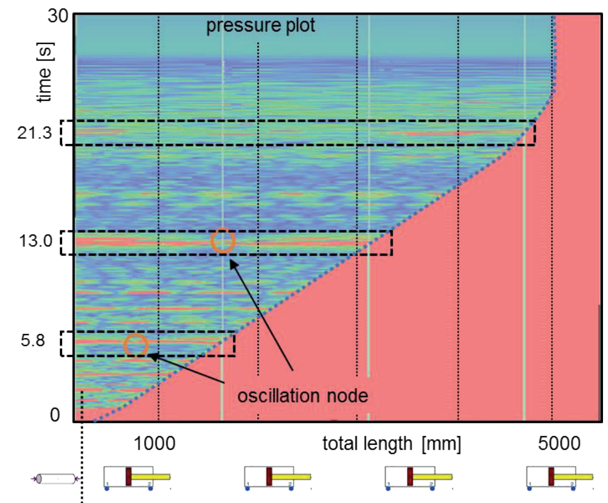

Figure 18: 2D pressure plot for model B

The direct comparison of all simulation results (mode A: figure 9 and model B: figure 17) impressively shows the immense difference between the classical and the new simulation model.

- different resonance frequencies
- resonance effects at different times and system states (cylinder positions)
- different amplitude of resonance effects

4 Summary and Conclusion

Hydraulic systems are widely used in stationery and mobile applications. A robust and secure application is possible, if no or only minimal resonance effects occur during operation. A fast and cost-effective development of complex systems is only achieved, if critical resonance situations are analysed, understood and eliminated in an early development state and especially before prototypes are build. Real applications showed that classical 1D simulation approaches are not capable to simulate all systems with the necessary accuracy. Especially in the case of hydraulic cylinders, which change the system resonance with movement, show no adequate results with classical concentrated model approaches. They predict wrong system behaviours and consequently misleading system resonance frequencies. Only a detailed consideration of all pressure wave effects inside moving cylinders allows a correct prediction of transient system resonance frequencies. This is achieved by an extension of accurate distributed parameter pipe models (method of characteristics), which allow variable transient pipe lengths.

Especially complex systems with multiple valves, pumps or cylinders feature different and continuous changing resonance and excitation frequencies. A static analysis of all possible valve position, cylinder position and pump speed combinations is extensive and typically not efficient (cost, time and resources). Additionally, transient effects of moving cylinders, pump speeds or valve switching are not considered, which prevent an assessment of the resonance effects in real application conditions. This leads wrong to conclusions and development targets.

Consequently, only the complete transient system analysis in combination with an accurate pressure wave consideration in all significant system parts enables a correct and constructive system simulation and analysis. This is the basis to invest money, time and resources at the right and essential system optimization tasks.

Nomenclature

Variable	*Description*	*Unit*
A	area	[m²]
c	speed of sound	[m/s]
D	diameter	[m]
f	frequency	[Hz]
L	length	[m]
n	speed	[1/min]
p	pressure	[bar]
Q	volumetric flow	[l/min]
t	time	[s]
λ	wave length	[m]

References

/1/ van Bebber, D., *Reduction of pressure waves in common rail systems by improving system design parameters.*, 5th International Fluid Power Conference, 5. IFK, Aachen, Germany, 2014.

/2/ Theissen, H., *Die Berücksichtigung instationärer Rohrströmung bei der Simulation hydraulischer Anlagen*, Dissertation IFAS, RWTH Aachen, 1984.

/3/ Müller, B., *Einsatz der Simulation zur Pulsations- und Geräuschminderung hydraulischer Anlagen,* Dissertation IFAS, RWTH Aachen, 2002.

/4/ Baum, H.; *Druckschwingungsanalyse hydraulischer Leitungssysteme – Eine Alternative zu CFD*, Digital Engineering Magazin 04-2016, Seite 28-30

/5/ Baum, H.; Scheffel, G.; Druckschläge im Gießaggregat von Druckgießmaschinen, Gießerei 104 08/2017, Seite 40-53

/6/ Baum, H., Pasquini, E.; *Druckschwingungsanalyse von Leitungssystemen mit viskoelastischem Materialverhalten der Rohrwand - Modellsynthese und messtechnische Ermittlung der Modellparameter*, VDI-Tagung SimVec, 22. - 23.11.2016, Baden-Baden

/7/ n. n.: *Typical measurements: axial piston pump*, http://people.bath.ac.uk/ensdnj/piston-pump.htm

/8/ Pasquini, E, Murrenhoff, H., Baum, H.; *Die Schallgeschwindigkeit von Fluiden in nachgiebigen Leitungen – Teil I: Leitungen mit linear-elastischem Materialverhalten*, O+P Fluidtechnik, 4/2017, Seite 66-71

/9/ Murrenhoff, H.; *Grundlager der Fluidtechnik – Teil 1: Hydraulik*, Vorlesungsumdruck IFAS, RWTH Aachen, 2012, Seite 217

/10/ Beater, P.; *Entwurf hydraulischer Maschinen*, ISBN 3-540-65444-5, VDI-Buch Springer-Verlag, 1999

/11/ Stulemeijer, I.; *Beschreibung der Druckschwankungen in hydrostatischen Anlagen und ihre Auswirkungen auf den Luftschall*, Dissertation TH Eindhoven, 1981, Seite 97

/12/ Baum, H. Eibl, S., Merck, J.; *Druckschwingungen in Kraftstoff-Niederdrucksystemen*, MTZ, 10/2017, Seite 62-66